Identification of Leakages in Water Networks
-Virtual Distortion Method Approach

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1. Abstract

A software tool for signal processing in health monitoring of water networks is presented. It is assumed that the water pressure in the network's nodes in a distance of inspected area can be measured and also that a diameter of some selected branch of the network can be modified in a controlled way. Then, making use of the analytical network model of this installation and using presented below, so called Virtual Distortion Method (VDM), the water leakage can be detected and identified (single as well as multiple locations and their intensities). The identification methodology takes advantage of gradient based optimization technique.

2. Key words
Leakage identification, water networks, inverse analysis

3. General characteristic

The problem of detection and identification of leakages (mostly due to corrosion) in water networks is an important issue. The problem of management of water sources is more and more important in the world scale. On the other hand, the consequences of unpredicted failure in the operating water network can be very grave. Therefore, there is requirement for an automatic monitoring system able to detect and localize leakages in the early stage of their development. The proposed approach is based on continuous observation of the pressure distribution in nodes of the water network. Having a reliable (verified versus field tests) numerical model of the network and its responses for determined inlet and outlet conditions, any modifications to the normal network response (pressure distribution) can be detected. Then, applying proposed bellow numerical procedure, the inverse problem of the water flow distribution can be performed. The possibility of simultaneous detection of several leakages with different locations and intensities is included into the proposed methodology.

The proposed methodology for the failure identification is based on so called *Virtual Distortion Method* (VDM) approach, applicable also in the problem of damage identification through monitoring of piezo-generated elastic wave propagation (Ref. 3). This technique (called *Piezodiagnostics*) is focused on efficient numerical performance of l inverse, non-linear, dynamic analysis. The crucial point of the concept is pre-computing of dynamic structural responses for locally generated impulse loadings by unit virtual distortions (similar to local heat impulses). These responses stored in so-called *influence matrix* allow considering all possible linear combinations of local perturbations (due to defect) and their influence on final structural response. Then, using a gradient-based optimization technique, the intensities of unknown, distributed virtual distortions (modeling local defects) can be tuned to minimize the distance between the computed final structural response and the measured one.

4. Definitions and linear analysis

Let us describe the *network analysis* (cf. Ref. 1) based approach to modelling of water systems using oriented *graf* of small example shown in Fig. 1, with topology defined by the following incidence matrix:

$$L=\begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{1}$$

where rows correspond to the network's nodes while columns correspond to the branches.

Defining the following quantities describing the state of the water network:

H – the vector of pressure (the height of water potential) in network's nodes
ε - the vector of drop of pressure in network's branches
Q – the vector of water flow in network's branches
R – the vector of flow capacity in network's branches (depends on pipes' cross-sections, length, material, etc.)
the following equations governing the water distribution can be formulated:

- equilibrium of inlets and outlets for nodes:

$$\mathbf{L\ Q = -q} \tag{2}$$

- definition of drops of pressure for branches

$$\mathbf{L}^{\mathbf{T}}\ \mathbf{H} = \mathcal{E} \tag{3}$$

- constitutive relation governing local flow in branches

$$\mathbf{Q} = -\mathbf{R}\ \mathcal{E} \tag{4}$$

where **q** denotes external inlet to the system.

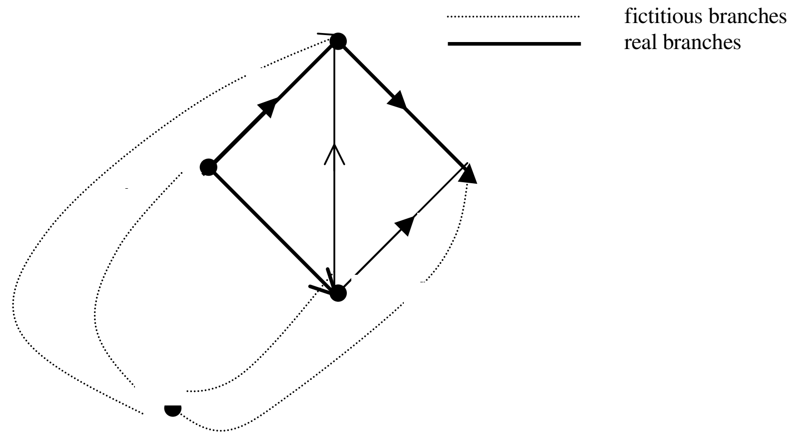

Fig. 1 Small water network scheme

The constitutive relation (4) is non-linear. Nevertheless, let us assume temporarily linearity of this relation. Substituting Eqs. (4) and (3) to (2), the following formula can be obtained:

$$\mathbf{L}\ \mathbf{R}\ \mathbf{L}^{\mathbf{T}}\ \mathbf{H} = \mathbf{q} \tag{5}$$

Describing the water network shown in Fig.1, the above set of equations takes the following form

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4' \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

where:

$$q_4 = R_4'(H_0 - H_4), \qquad R = \frac{K^2}{l},$$

K- the characteristic of the element,
l - the element's length,
H – denotes the water pressure in the node (*height* of water)
q – denotes the flow in the branch,

and it was assumed that the network is supplied only through the node No.1 (inlet with intensity q_1) and the only outlet is through the node No.4 (the coefficient R_4' =1). $R_2' = R_3' = 0$, which means, that the outlets in nodes No.2 and 3 vanish.

Assuming the following data for the small network (R_1=0.004, R_2=R_3=R_5=0.016, R_4=0.011, q_1=0.05 m^3/sec, H_0=0.0):

$$\begin{bmatrix} 0.02 & -0.004 & -0.016 & 0 \\ -0.004 & 0.031 & -0.011 & -0.016 \\ -0.016 & -0.011 & 0.043 & -0.016 \\ 0 & -0.016 & -0.016 & 1.032 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

results in the following distribution of water heights: $\mathbf{H}^q$ = [4,3124, 1,2794 1,9456 0,0500],

5. Modelling of non-linearities by virtual distortions

Local non-linearity can be included into the system through so called *virtual distortion* (cf. Ref. 2) $\boldsymbol{\varepsilon}^{\mathbf{o}}$ introduced into the formula (5):

$$\mathbf{LR}\ (\ \mathbf{L}^{\mathbf{T}}\ \mathbf{H} - \boldsymbol{\varepsilon}^{\mathbf{0}}\ ) = \mathbf{q} \tag{5a}$$

The influence of virtual distortions on the resultant flow redistribution can be calculated making use of so called *influence matrix* D^H_{ij} describing water pressure H^R_i induced in the network as the response for the unit virtual distortion $\varepsilon^o_j=1$ generated in the branch *j*. Therefore, the vector $\mathbf{H}^R$ can be calculated from the following equation obtained from Eq.5a:

$$\mathbf{LR\,L^T\,H^R = LRI}, \qquad (8)$$

Coming back to our small example, let us generate the unit virtual distortion in the branch No. 4. The corresponding set of equations (8) with modifications due to the boundary conditions (cf. Eq. 6) takes the following form:

$$\begin{bmatrix} 0{,}02 & -0{,}004 & -0{,}016 & 0 \\ -0{,}004 & 0{,}0313 & -0{,}0113 & -0{,}016 \\ -0{,}016 & -0{,}0113 & 0{,}0433 & -0{,}016 \\ 0 & -0{,}016 & -0{,}016 & 1{,}032 \end{bmatrix} \begin{bmatrix} H_1^{R'} \\ H_2^{R'} \\ H_3^{R'} \\ H_4^{R'} \end{bmatrix} = - \begin{bmatrix} 0 \\ -0{,}0113\,\varepsilon_4^0 \\ 0{,}0113\,\varepsilon_4^0 \\ 0 \end{bmatrix} \qquad (9)$$

where $\varepsilon_4^0=1$.

The resulting distribution of potentials is: $\mathbf{H}^R=[0.1508\ \ -0{,}2513\ \ 0.2513\ \ 0.0]^T$
Applying the same procedure to other branches, the following influence matrix can be determined:

$$D^H = \begin{bmatrix} .2426401350 & .7573598650 & .3933996624 & -.1507595274 & .6066003376 \\ -.07106689171 & .07106689171 & .6776672293 & .2512658790 & .3223327707 \\ .07106689171 & -.07106689171 & .3223327707 & -.2512658790 & .6776672293 \\ 0.0000000000 & 0.0000000000 & 0.0000000000 & 0.0000000000 & 0.0000000000 \end{bmatrix}$$

6. Water network diagnostics – problem formulation

The virtual distortion can also simulate leakage from the water network and therefore, can be useful in identification of this type of damage. For example, a leakage from the branch no. 4 of our testing network (Fig. 2) can be modelled (with respect to a scalar coefficient) through the solution of the following problem (cf. Eqs. 9):

$$\begin{bmatrix} 0{,}02 & -0{,}004 & -0{,}016 & 0 \\ -0{,}004 & 0{,}0313 & -0{,}0113 & -0{,}016 \\ -0{,}016 & -0{,}0113 & 0{,}0433 & -0{,}016 \\ 0 & -0{,}016 & -0{,}016 & 1{,}032 \end{bmatrix} \begin{bmatrix} H_1^{R'} \\ H_2^{R'} \\ H_3^{R'} \\ H_4^{R'} \end{bmatrix} = - \begin{bmatrix} 0 \\ 0{,}0113\,\varepsilon_4^0 \\ 0{,}0113\,\varepsilon_4^0 \\ 0 \end{bmatrix} \qquad (10)$$

where $\varepsilon_4^0=1$. Note, that the compensative load (blue arrows in Fig. 2) models inlet or outlet from the branch No.4. The resulting distribution of potentials is $\mathbf{H}^R=-[0.72975\ \ 0{,}72975\ \ 0.72975\ \ 0.02263]^T$.

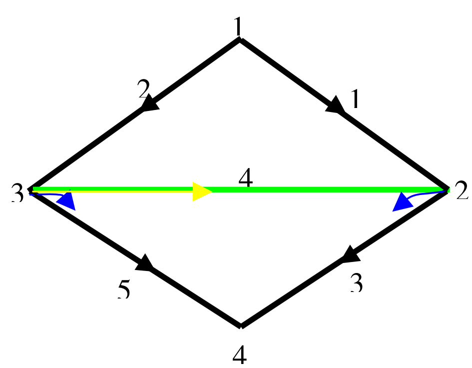

Fig. 2 Small water network with no leakage

Applying the same procedure to other branches, the following influence matrix can be determined:

$$\tilde{D}^{H} = \begin{bmatrix} 0,44730 & 2,00256 & 0,42540 & 0,72975 & 0,63860 \\ 0,27577 & 0,24773 & 0,70967 & 0,72975 & 0,35433 \\ 0,24023 & 1,31627 & 0,35433 & 0,72975 & 0,70967 \\ 0,00800 & 0,03200 & 0,03200 & 0,02263 & 0,03200 \end{bmatrix} \qquad (11)$$

Let us now formulate the leakage identification problem. Assuming a leakage in the branch No. 4 of the network shown in Fig. 2, let us model this situation adding a node No. 5 (cf. Fig. 3). The corresponding flow distribution can be described by the following set of equations (cf. Eq. 6):

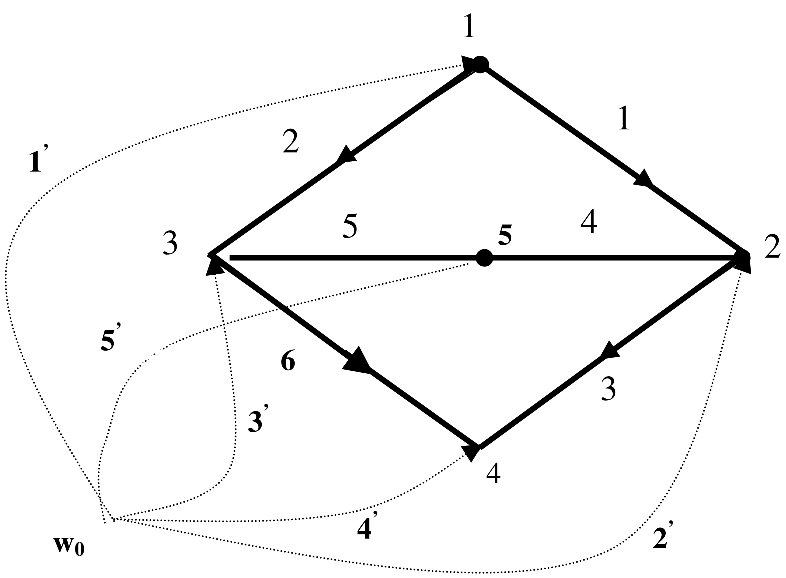

Fig. 3 Modelling of network with one leakage

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 & 0 \\ -R_1 & R_1+R_3+R_4 & 0 & -R_3 & -R_4 \\ -R_2 & 0 & R_2+R_5+R_6 & -R_5 & -R_6 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4' & 0 \\ 0 & -R_4 & -R_6 & 0 & R_4+R_6+R_5' \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (12)$$

where the following formulas have been applied:

$$q_4 = R_4'(H_0 - H_4) \quad \text{and} \quad q_5 = R_5'(H_0 - H_5),$$

and it has been assumed, that the only inlet is applied in the node No.1, while two outlets in nodes No. 4 and No. 5 are determined by their resistances : R_4 and R_5. Assuming also: $R_2 = R_3 = 0$ and $R_i=0.016$ (i=6-9), $R_5=0,004$, $q_1= 0.05$ m^3/s and $H_0 = 0$, the above set of equations takes the form:

$$\begin{bmatrix} 0,02 & -0,004 & -0,016 & 0 & 0 \\ -0,004 & 0,0426 & 0 & -0,016 & -0,0226 \\ -0,016 & 0 & 0,0546 & -0,016 & -0,0226 \\ 0 & -0,016 & -0,016 & 1,032 & 0 \\ 0 & -0,0226 & -0,0226 & 0 & 1,0452 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} = \begin{bmatrix} 0,05 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (13)$$

and leads to the solution $H^{q'}_i$= [3,37274 0,33976 1,00599 0,02086 0.02914]. It can be demonstrated, that the above result is equal (taking into account only the first four nodes) to the following linear combination of previously determined solutions:

$\mathbf{H}^{q'} = \mathbf{H}^q$ - 1.2876 $\mathbf{H}^{\mathbf{R}}$. Therefore, the solution of the set of equations (13) can be optionally described as the solution of the following optimisation problem:

$$\mathbf{min\ f = min} \sum_i [(\mathbf{H_i} - \mathbf{H_i^{q'}})^2] \tag{14}$$

where

$$\mathbf{H} = \mathbf{H}^q + \mathbf{D}^\mathbf{H} \varepsilon_4^0 \quad \geq \mathbf{0} \tag{15}$$

and:

- $\mathbf{H}^{q'}$ denotes the water pressure distribution measured in the network
- $\mathbf{H}^q$ denotes the water pressure distribution calculated for the initial network configuration
- ε_4^0 denotes the unknown coefficient simulating water leakage in the branch No.4.

subject to the following constraints:
Generalising the above formulation, we can search for leakages in all branches. In this case, the equation (15) should be replaced by the following condition:

$$H_i = H^q_i + \Sigma_j D^H_{ij} \varepsilon_j^0 \tag{16}$$

where five unknowns ε_j^0 describe potential leakage intensities and the influence matrix $\mathbf{D}^\mathbf{H}$ is determined by Eq. 11.

7. Numerical example
Let us consider the water network shown in Fig. 2. The heights of the nodes for the network in its natural state, assuming linear constitutive relation, are as follows:
$\mathbf{H}^q$ = [4,3124, 1,2794 1,9456 0,0500],
The measured heights of the nodes for the network exhibiting a leakage are as follows:
H_i^p= [3,37274 0,33976 1,00599 0,02086 0.02914].
The QP solver (cf. Ref. 4) has been employed to find the solution of the leakage identification problem posed by (14), (16).
One leakage has been identified and the value of the distortion modelling this leakage is $\varepsilon_4^0 = -1.2876$.
The development of five unknown virtual distortions, supposed to model leakages, is shown in Fig. 4.

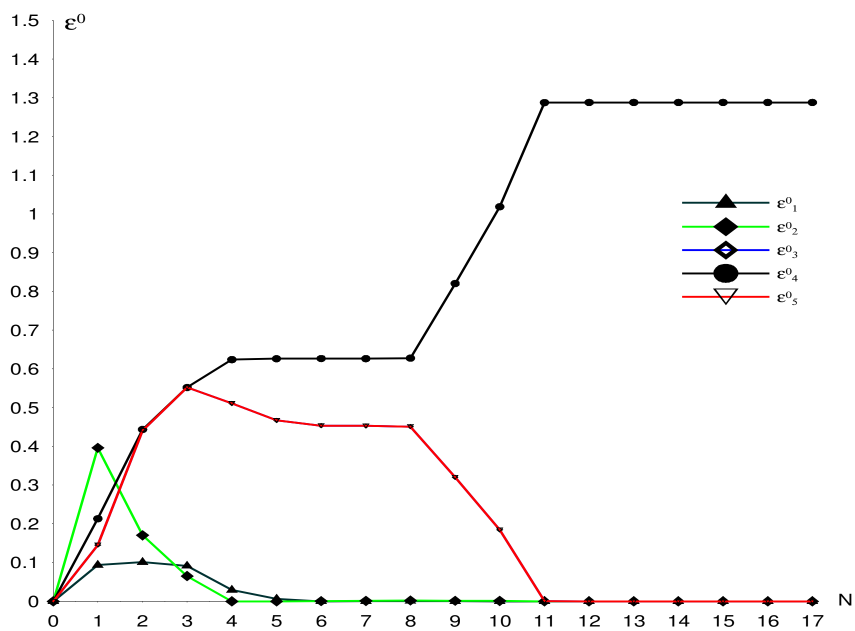

Fig. 4 Virtual distortions development in the optimisation process for linear case

In the non-linear case we have analysed the non-linearity of the flow/drop-of-pressure relation through superposition of the two following virtual distortion fields: the first one D^H ($\beta^0 = 1$), modelling physical (constitutive) non-linearities and the second one $\tilde{D}^H$ ($\varepsilon^0 = 1$), describing leakages.
The height of the nodes for the network in its natural state are as follows:
$\mathbf{H}$ = [4,3124, 1,2794 1,9456 0,0500],

while the measured height of the nodes for the damaged network are as follows:
H_i^p= [3,37274 0,33976 1,00599 0,02086 0.02914].
The value of the distortions modelling physical non-linearity are as follows:

$$\beta_j^0 = [0.3079 \ \ -0.3079 \ \ 0.0000 \ \ 0.0000 \ \ 0.0000]$$

and the value of the distortions modelling the leakage are as follows:

$$\varepsilon_j^0 = [0.0000 \ \ 0.0000 \ \ 0.0000 \ \ -1.2876 \ \ 0.0000]$$

The development of five unknown virtual distortions, supposed to model leakages, is shown in Fig. 5.

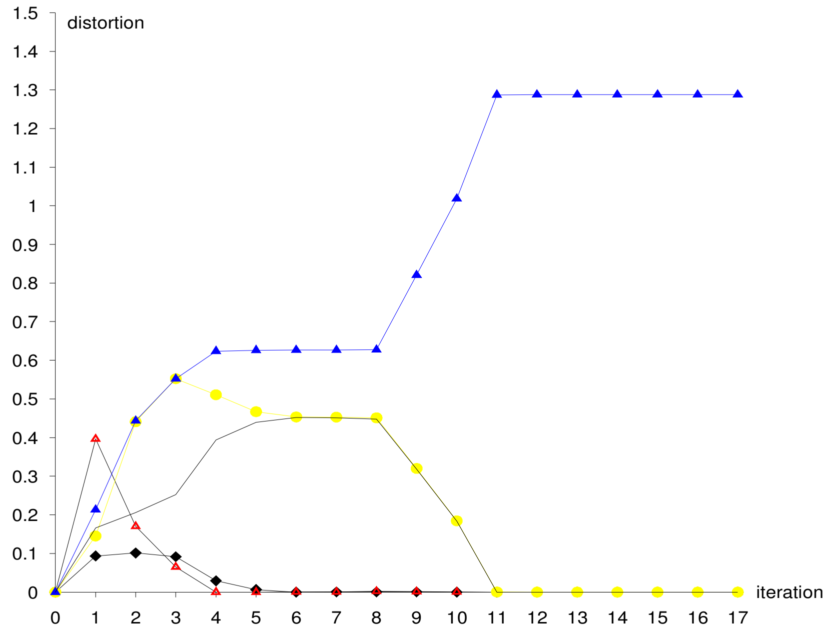

Fig. 4 Virtual distortions development in the optimisation process for non-linear case

8. Conclusions

It has been demonstrated, that having numerical model of the water network and knowing nodal pressure distribution (measured in real time), current inlets and outlets, the distribution of possible several simultaneous leakages in branches and their intensities can be determined. We can expect precise leakage identification if the number of measurements is not lower than the number of assumed all possible locations of leakages.

Linear constitutive relations has been assumed in the first formulation. Then, the non-linearity of the flow/drop-of-pressure relation has been taken into account through superposition of the following two virtual distortion fields: the first one, modelling physical non-linearities and the second one, describing leakages.

9. Acknowledgements

The authors would like to gratefully acknowledge the financial support through the 5FP EU project Research Training Networks "SMART SYSTEMS" HPRN-CT-2002-00284 and through the grant No. KBN 5T07A05222 funded by the State Committee for Scientific Research in Poland. The work presents a part of the Ph.D. thesis of the third author, supervised by the first author.

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